to B. If k_2 is even the cord passed last to the pencil; then pass it around the point C making k_3 plies between P and C, and attach the end to the pencil or to the peg at C according as k_3 is even or odd. But if k_2 is odd, the cord passed last to the peg at B; then let it pass from B to C, and then from C around the pencil at P until the requisite number, k_3, of plies extend from C to P; finally attach the end to the pencil or to the peg at C according as k_3 is odd or even.

Now let both cords be stretched tight while the pencil is held firmly at P. Then tie the free ends of the cords together at some convenient distance from the pencil so that when a pull is made on the knot both strings will be drawn tight throughout their entire lengths, with the exception of course of the free ends beyond the knot. Then if the pencil moves and the cords are kept always in the position which has been defined, it is evident that the pencil point describes the branch in consideration; for k_1 times the distance from A remains always equal to k_2 times the distance from B plus k_3 times the distance from C.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

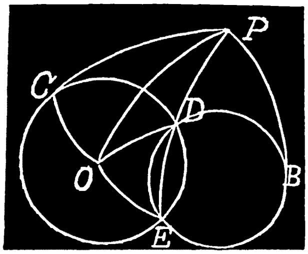
GEOMETRY.

20. Proposed by DR. GEORGE BRUCE HALSTED, Greeley, Colo.

Demonstrate by pure spherical geometry that spherical tangents from any point in the produced spherical chord common to two intersecting circles on a sphere are equal.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

No solution of this problem has yet appeared in the MONTHLY. A simple geometrical solution such as is possible for the corresponding problem in planes is not possible for this problem. The following solution is quite simple.



Let P be point on the common chord DE; PB, PC the tangents, O the pole of one circle. Let $PE=R$, $PD=r$, $PC=\rho$, $PB=\rho'$, $PO=\delta$, $OD=OE=OC=\beta$, $\angle EPO=\phi$.

Then $\cos\beta=\cos R\cos\delta+\sin R\sin\delta\cos\phi \ldots (1)$,

$\cos\beta=\cos r\cos\delta+\sin r\sin\delta\cos\phi \ldots (2)$,

$\cos\delta=\cos\beta\cos\rho \ldots (3)$,

$\cos\phi$ from (1) in (2) gives $\cos\beta(\sin R-\sin r)=\cos\delta\sin(R-r) \ldots (4)$.

$\cos\delta$ from (3) in (4) gives $\cos\rho=(\sin R-\sin r)/\sin(R-r)$.

Similarly, $\cos\rho'=(\sin R-\sin r)/\sin(R-r)$. $\therefore\rho=\rho'$.
These equations reduce to $\tan^2\frac{1}{2}\rho=\tan^2\frac{1}{2}\rho'=\tan\frac{1}{2}R\tan\frac{1}{2}r$.

Professor Philbrick gave a solution of this problem at the time it was published but it did not fill the requirements because it was not a pure spherical geometry solution.

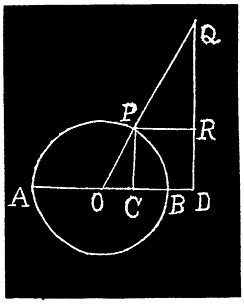
290. Proposed by J. J. QUINN, Scottdale, Pa.

(a) Suppose a circle described around the origin. Then at the end of a uniformly revolving radius r, a line equal to the diameter is pivoted. Find the equation of the locus of its extremity, if for every unit of angle its projection on the X axis is a constant linear unit, being the same part of the diameter as the angle is of π radians.

(b) Show how it can be applied to the trisection or multisection of an angle.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

(a) Let angle $POB=\theta$. Then CD, the projection of $PQ=2r$ on AB, is $r\,\theta/90$. $OD=x=r\cos\theta+r\,\theta/90$.



$DQ=y=r\sin\theta+r\sqrt{[4-(\theta/90)^2]}$.

$\rho^2=x^2+y^2=5r^2+2r^2\cos\theta(\theta/90)+2r^2\sin\theta\sqrt{[4-(\theta/90)^2]}$ is the polar equation sought.

(b) Let $m\phi$ be the angle to be multisected. $m\phi/m=\phi$. Lay off $OD=x=r\cos\phi+r\,\phi/90$.

Then erect $DQ=y=r\sin\phi+r\sqrt{[4-(\phi/90)^2]}$ perpendicular to OD at D. From Q as center, with radius equal to $2r$ describe an arc cutting the circumference of the given circle at P.

Draw PO; then $\angle POD=\phi$.

300. Proposed by J. J. QUINN, Ph. D., Scottdale, Pa.

Trisect an angle by means of a tractrix.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philahelphia, Pa.

The length of the tangent between the axis of abscissas and the point of tangency is constant.

Let a=length of this tangent, y=an ordinate opposite angle θ, z=an ordinate opposite angle ϕ. Also let $\theta=3\phi$.

$\therefore y=a\sin\theta=3a\sin\phi-4a\sin^3\phi$, $z=a\sin\phi$.

$\therefore y/z=3-4\sin^2\phi$ or $\sin\phi=\frac{1}{2}\sqrt{[(3z-y)/z]}$.

$\therefore z^2/a^2=\dfrac{3z-y}{4z}$ or $y=\dfrac{(3a^2-4z^2)z}{a^2}$.

Let $PD=z$, $PCD=\angle\phi$. Construct $QB=y=\dfrac{(3a^2-4z^2)z}{a^2}$.

Let $QAB=\theta$ where $PC=QA=a$. Then $\theta=3\phi$.

$\therefore$ Parallel to PC draw AR, then $\angle RAB=\frac{1}{3}\angle QAB$.

$PD=z$ cannot be greater than $\frac{1}{2}a$, then $y=a$, $\theta=\frac{1}{2}\pi$, $\phi=\frac{1}{6}\pi$.